Protection Basics: Introduction to Symmetrical Components

1. Introduction

Symmetrical components is the name given to a methodology, which was discovered in 1913 by Charles Legeyt Fortescue who later presented a paper on his findings entitled, "Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks." Fortescue demonstrated that any set of unbalanced three-phase quantities could be expressed as the sum of three symmetrical sets of balanced phasors. Using this tool, unbalanced system conditions, like those caused by common fault types may be visualized and analyzed. Additionally, most microprocessorbased relays operate from symmetrical component quantities and so the importance of a good understanding of this tool is self-evident.

2. Positive, Negative and Zero Sequence Components

According to Fortescue's methodology, there are three sets of independent components in a three-phase system: positive, negative and zero for both current and voltage. Positive sequence voltages (Figure 1) are supplied by generators within the system and are always present. A second set of balanced phasors are also equal in magnitude and displaced 120 degrees apart, but display a counter-clockwise rotation sequence of A-C-B (Figure 2), which represents a negative sequence. The final set of balanced phasors is equal in magnitude and in phase with each other, however since there is no rotation sequence (Figure 3) this is known as a zero sequence.



Figure 1. Positive Sequence Components; A-B-C







3. Introduction to Symmetrical Components

The symmetrical components can be used to determine any unbalanced current or voltage (Ia, Ib, Ic or Va, Vb, Vc which reference unbalanced line-to-neutral phasors) as follows:

$I_{0} = I_{1} + I_{2} + I_{0}$	$V_{a} = V_{1} + V_{2} + V_{0}$
$I_{b} = a^{2}I_{1} + aI_{2} + I_{0}$	$V_{b} = a^{2}V_{1} + aV_{2} + V_{0}$
$I_{c} = aI_{1} + a^{2}I_{2} + I_{0}$	$V_{c} = aV_{1} + a^{2}V_{2} + V_{0}$

The sequence currents or voltages from a three-phase unbalanced set can be calculated using the following equations:

Zero Sequence Component:

$$I_0 = \frac{1}{3} (I_a + I_b + I_c)$$
 $V_0 = \frac{1}{3} (V_a + V_b + V_c)$

Positive Sequence Component:

 $I_1 = \frac{1}{3} (I_0 + aI_b + a^2 I_c)$ $V1 = \frac{1}{3} (V_0 + aV_b + a^2 V_c)$

Negative Sequence Component:

$$I_2 = \frac{1}{3} (I_a + a^2 I_b + a I_c) V_2 = \frac{1}{3} (V_a + a^2 V_b + a V_c)$$

The independence of the symmetrical components and their resultant summation follow the principle of superposition, which is the basis for its practical usage in protective relaying.

Before proceeding further, a mathematical explanation of the "a" operator is required. Within Fortescue's formulas, the "a" operator shifts a vector by an angle of 120 degrees counter-clockwise, and the "a^{2"} operator performs a 240 degrees counter-clockwise phase shift. According to Fortescue, a balanced system will have only positive sequence currents and voltages. For example, as

shown in Figure 4, the calculation of symmetrical components in a three-phase balanced or symmetrical system results in only positive sequence voltages, $3V_1$. Similarly, the currents also have equal magnitudes and phase angles of 120 degrees apart, which would produce a result of only positive sequence and no negative or zero sequence currents for a balanced system.

For unbalanced systems, such as an open-phase there will be positive, negative and possibly zero-sequence currents. Referring to the open-phase example in Figure 5, it can be seen that the calculation of the symmetrical components results in positive, negative and zero sequence currents of $3I_1$, $3I_2$, and $3I_0$. However, since the voltages are balanced in magnitude and phase angle, the result would be the same as the balanced system in Figure 4, which produces only positive sequence voltage.

Similarly for a single phase to ground fault as shown in Figure 6, there will be positive, negative and zero sequence currents $(3I_1, 3I_2, and 3I_0)$ and voltages $(3V_1, 3V_2, and 3V_0)$.

4. Summary

Under a no fault condition, the power system is considered to be essentially a symmetrical system and therefore only positive sequence currents and voltages exist. At the time of a fault, positive, negative and possibly zero sequence currents and voltages exist. Using real world phase voltages and currents along with Fortescue's formulas, all positive, negative and zero sequence currents can be calculated. Protective relays use these sequence components along with phase current and/or voltage data as the input to protective elements.



Figure 4. Three-Phase Balanced / Symmetrical System

5. References

[1] J. L. Blackburn, T. J. Domin, 2007, "Protective Relaying, Principles and Applications, Third Edition," Taylor & Francis Group, LLC, Boca Raton, FL, pp.75-80

[2] GE Publication, "Fundamentals of Modern Protective Relaying," Instruction Manual, Markham, Ontario, 2007



Figure 5. Open-Phase Unbalanced / Non-Symmetrical System



Figure 6. Single Phase-Ground Fault Unbalanced / Non-Symmetrical System