9 METHODS FOR ANALYZING GENERALIZING, AND VISUALIZING RELAY RESPONSE

The material that has been presented thus far will enable one to translate power-system currents and voltages into protective-relay response in any given case. From that standpoint, the material of this chapter is unnecessary. Nor is this chapter intended to teach one how to determine these currents and voltages by the methods of symmetrical components,[1,2] since it is assumed that this is known. The purpose of this chapter is best explained by a simple example.

In Chapter 7, we learned that a relay coil connected in the neutral lead of three wye-connected current transformers would have a current in it equal to $3I_{a0}$. Assuming that this is an overcurrent relay, we can immediately say that this relay will respond only to zero-phase-sequence current. This is important and useful knowledge, because we then know that the relay will respond only to faults involving ground. Furthermore, we do not have to calculate the positive- and negative-phase-sequence components of current in the circuit protected by the relay; all we need to know is the zero-phase-sequence component. Moreover, merely by looking at the phase-sequence diagram for any fault, we can tell whether this relay will receive zero-phase-sequence current, and how the magnitude and direction of this current will change with a change in fault location. Therefore, it is evident that we have at our disposal a much broader conception of the response of this relay than merely knowing that it will operate whenever it receives more than a certain magnitude of current. The value of being able to visualize and generalize relay response will become even more evident in the case of any relay that responds to certain combinations of voltage, current, and phase angle.

THE *R-X* DIAGRAM

The *R-X* diagram was introduced in Chapter 4 to show the operating characteristics of distance relays. Now, we are about to use it to study the response of distance-type relays to various abnormal system conditions. With this diagram, the operating characteristic of any distance relay can be superimposed on the same graph with any system characteristic, making the response of the relay immediately apparent.

A distance relay operates for certain relations between the magnitudes of voltage, current, and the phase angle between them. For any type of system-operating condition, there are certain characteristic relations between the voltage, current, and phase angle at a given distance-relay location in the system. Thus, the procedure is to construct a graph showing the relations between these three quantities (1) as supplied from the system, and (2) as required for relay operation.

PRINCIPLE OF THE R-X DIAGRAM

As described in Chapter 4, the basis of the *R-X* diagram is the resolution of the three variables–voltage, current, and phase angle–into two variables. This is done by dividing the rms magnitude of voltage by the rms magnitude of current and calling this an impedance *"Z."*

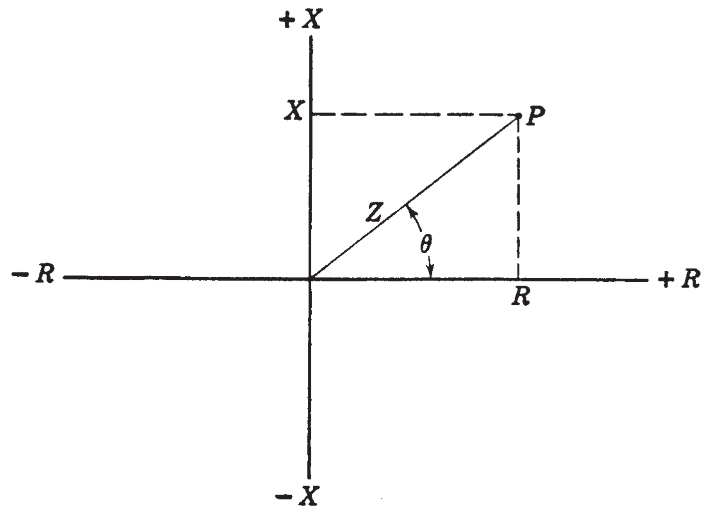

Fig. 1. The *R-X* diagram.

(For the moment, let us not be concerned with the significance of this impedance.) Then, the resistance and reactance components of *Z* are derived, by means of the familiar relations $R = Z \cos \theta$ and $X = Z \sin \theta$. We shall call θ positive when *I* lags *V,* assuming certain relay connections and references. These values of *R* and *X* are the coordinates of a point on the *R-X* diagram representing a given combination of *V, I,* and θ. *R* and *X* may be positive or negative, but *Z* must always be positive; any negative values of *Z* obtained by substituting certain values of θ in an equation should be ignored since they have no significance.

Figure 1 shows the *R-X* diagram and a point *P* representing fixed values of *V, I,* and θ, when *I* is assumed to lag *V* by an angle less than 90°. A straight line drawn from the origin to *P* represents *Z,* and θ is the angle measured counterclockwise from the + *R* axis to *Z.*

It is probably obvious that *P* can be located from a knowledge of *Z* and θ without deriving the *R* and *X* components. Or, by calculating the complex ratio of *V* to *I,* the values of *R* and *X* can be obtained immediately without consideration of θ. If *V, I,* and θ vary, several points can be plotted, and a curve can be drawn through these points to represent the characteristic.

In order to superimpose the plot of a relay characteristic on the plot of a system characteristic to determine relay operation, both plots must be on the same basis. A given relay operates in response to voltage and current obtained from certain phases.[3] Therefore, the system characteristic must be plotted in terms of these same phase quantities as they exist at the relay location. Also, the coordinates must be in the same units. The per unit or percent system is generally employed for this purpose. If actual ohms are used, both the power-system and the relay characteristics must be on either a primary or a secondary basis, taking into account the current- and voltage-transformation ratios, as follows:

$$\text{Secondary ohms} = \text{Primary ohms} \times \frac{\text{CT ratio}}{\text{VT ratio}}$$

Finally, both coordinates must have the same scale, because certain characteristics are circular if the scales are the same.

It is necessary to establish a convention for relating a relay characteristic to a system characteristic on the *R-X* diagram. The convention must satisfy the requirement that, for a system condition requiring relay operation, the system characteristic must lie in the operating region of the relay characteristic. The convention is to make the signs of *R* and *X* positive when power and lagging reactive power flow in the tripping direction of the distance relays under balanced three-phase conditions. Lagging reactive power is here considered to flow in a certain direction when current flows in that direction as though into a load whose reactance is predominantly inductive. It is the practice to assume “delta” (defined later) currents and voltages as the basis for plotting both system and relay characteristics when phase distance relays are involved, or phase current and the corresponding phase-to-ground voltage (called “wye” quantities) when ground distance relays are involved. In either case, three relays are involved, each receiving current and voltage from different phases. Either the delta or the wye voltage-and-current combinations will give the same point on the *R-X* diagram under balanced three phase conditions.

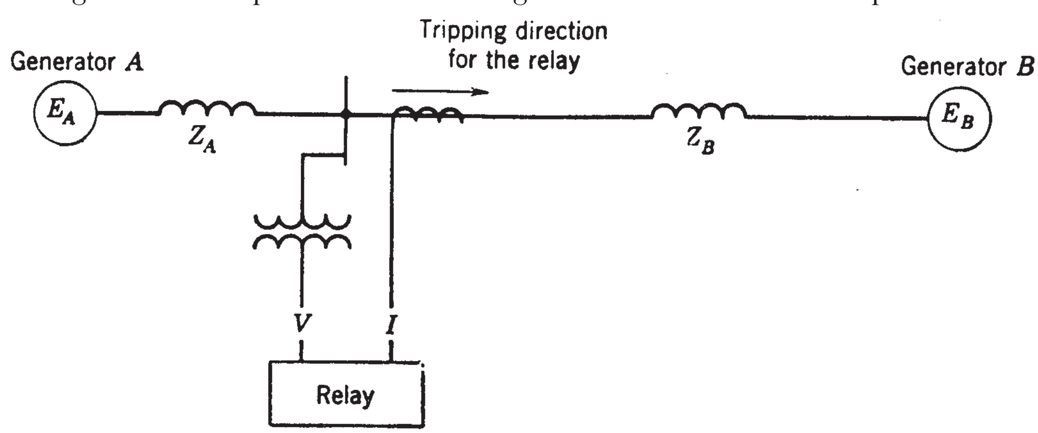

Fig. 2. Illustration for the convention for relating relay and system characteristics on the *R-X* diagram.

To illustrate this convention, refer to Fig. 2 where a distance relay is shown to be energized by voltage and current at a given location in a system. The coordinates of the impedance point on the *R*-*X* diagram representing a balanced three-phase system condition as viewed in the tripping direction of the relay will have the signs as shown in Table 1. Leading reactive power is here considered to flow in a certain direction when current flows in that direction as though into a load whose reactance is predominantly capacitive.

Table 1. Conventional Signs of *R* and *X*

Condition	Sign of *R*	Sign of *X*
Power from *A* toward *B*	+	
Power from *B* toward *A*	–	
Lagging reactive power from *A* toward *B*		+
Lagging reactive power from *B* toward *A*		–
Leading reactive power from *A* toward *B*		–
Leading reactive power from *B* toward *A*		+

The following relations give the numerical values of *R* and *X* for any balanced three-phase condition:

$$R = \frac{V^2 W}{W^2 + RVA^2}$$

$$X = \frac{V^2 RVA}{W^2 + RVA^2}$$

where V is the phase-to-phase voltage, W is the three-phase power, and RVA is the three-phase reactive power. R and X are components of the positive-phase-sequence impedance which could be obtained under balanced three-phase conditions by dividing any phase-to-neutral voltage by the corresponding phase current. All the quantities in the formulae must be expressed in actual values (i.e., ohms, volts, watts, and reactive volt-amperes), or all in percent or per unit.

By applying the proper signs to *R* and *X*, one can locate the point on the *R*-*X* diagram representing the impedance for any balanced three phase system condition. For example, the point *P* of Fig. 1 would represent a condition where power and lagging reactive power were being supplied from *A* toward *B* in the tripping direction of the relay.

For a relay in the system of Fig. 2 whose tripping direction is opposite to that shown, interchange *A* and *B* in the designation of the generators of Fig. 2 and in Table 1; in other words, follow the rule already given that the signs of *R* and *X* are positive when power and lagging reactive power flow in the tripping direction of the relay. For example, if the point *P* of Fig. 1 represents a given condition of power and reactive power flow as it appears to the relay of Fig. 2, then to a relay with opposite tripping direction the same condition appears as a point diametrically opposite to *P* on Fig. 1. Occasionally, it may be desired to show on the same diagram the characteristics of relays facing in opposite directions. Then the rule cannot be followed, and care must be taken to avoid confusion.

Because it is customary to think of impedance in terms of combinations of resistance and reactance of circuit elements, one may wonder what significance the Z of an R-X diagram has. By referring to Fig. 2, it can easily be shown that the ratio of V to I is as follows:

$$\frac{V}{I} = Z = \frac{E_A Z_B + E_B Z_A}{E_A - E_B}$$

where all quantities are complex numbers, and where E_A and E_B are the generated voltages of generators A and B, respectively. Therefore, in general, Z is not directly related to any actual impedance of the system. From this general equation, system characteristics can be developed for loss of synchronism between the generators or for loss of excitation in either generator.

For normal load, loss of synchronism, loss of excitation, and three phase faults–all balanced three-phase conditions–a system characteristic has the same appearance to each of the three distance relays that are energized from different phases. For unbalanced short circuits, the characteristic has a different appearance to each of the three relays, as we shall see shortly.

Other conventions involved in the use of the R-X diagram will be described as it becomes necessary.

By using distance-type relay units individually or in combination, any region of the R-X diagram can be encompassed or set apart from another region by one or more relay characteristics. With the knowledge of the region in which any system characteristic will lie or through which it will progress, one can place distance-relay characteristics in such a way that a desired kind of relay operation will be obtained only for a particular system characteristic.

SHORT CIRCUITS

For general studies, it is the practice to think of a power system in terms of a two-generator equivalent, as in Fig. 3. The generated voltages of the two generators are assumed to be equal and in phase. The equivalent impedances to the left of the relay location and to the

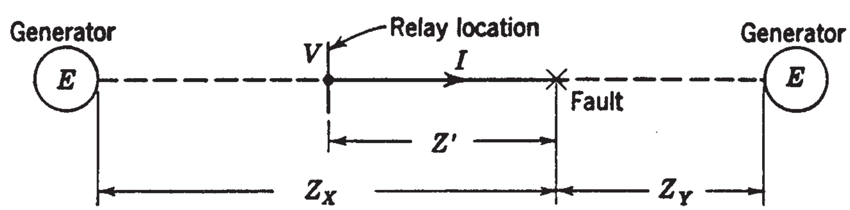

Fig. 3. Equivalent-system diagram for defining relay quantities during faults.

right of the short circuit are those that will limit the magnitudes of the short-circuit currents to the actual known values. The short circuit is assumed to lie in the tripping direction of the relay.

The possible effect of mutual induction from a circuit paralleling the portion of the system between the relay and the fault will be neglected. Also, load and charging current will be neglected; however, they may not be negligible if the fault current is very low.

Nomenclature to identify specific values or combinations of the quantities indicated on Fig. 3 will be as follows:

Z = System impedance viewed both ways from the fault = $\dfrac{Z_X Z_Y}{Z_X + Z_Y}$

C = Ratio of the relay current I to the total current in the fault = $\dfrac{Z_Y}{Z_X + Z_Y}$

Subscripts *a, b,* and *c* denote phases *a, b,* and *c,* respectively. Throughout this book, positive phase sequence is assumed to be *a-b-c*. Subscripts 1, 2, and 0 denote positive, negative, and zero phase sequence, respectively.

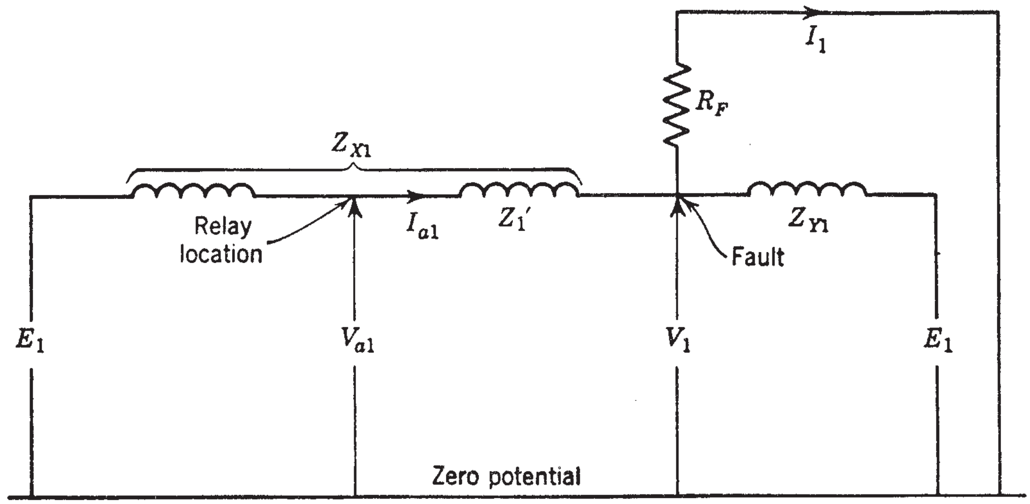

Fig. 4. Positive-phase-sequence network for a three-phase fault.

THREE-PHASE SHORT CIRCUITS

For a three-phase fault, the positive-phase-sequence network is as shown in Fig. 4 for the quantities of phase *a*. Whenever the term "three-phase" fault is used, it will be assumed that the fault is balanced, i.e., that only positive-phase-sequence quantities are involved. The quantity R_F is the resistance in the short circuit, assumed to be from phase to neutral of each phase.

By inspection, we can write:

$$I_1 = \frac{E_1}{Z_1 + R_F}$$

$$I_{a1} = \frac{Z_{Y1} I_1}{Z_{Y1} + Z_{X1}} = C_1 I_1 = \frac{C_1 E_1}{Z_1 + R_F}$$

$$V_1 = I_1 R_F = \frac{E_1 R_F}{Z_1 + R_F}$$

$$V_{a1} = V_1 + I_{a1} Z_1' = \frac{E_1 R_F}{Z_1 + R_F} + \frac{C_1 E_1 Z_1'}{Z_1 + R_F}$$

Let
$$\frac{E_1}{Z_1 + R_F} = \frac{1}{K}$$

Then
$$KI_{a1} = C_1$$

$$KV_{a1} = R_F + C_1 Z_1'$$

Since there are no negative-or zero-phase-sequence quantities for a three-phase fault, we can write:

$$KI_{a2} = 0$$

$$KI_{a0} = 0$$

$$KV_{a2} = 0$$

$$KV_{a0} = 0$$

Therefore, the actual phase currents and phase-to-neutral voltages at the relay location are:

$$KI_a = KI_{a1} + KI_{a2} + KI_{a0} = C_1$$

$$KI_b = a^2 KI_{a1} + aKI_{a2} + KI_{a0} = a^2 C_1$$

$$KI_c = aKI_{a1} + a^2 KI_{a2} + KI_{a0} = aC_1$$

$$KV_a = KV_{a1} + KV_{a2} + KV_{a0} = R_F + C_1 Z_1'$$

$$KV_b = a^2 KV_{a1} + aKV_{a2} + KV_{a0} = a^2 (R_F + C_1 Z_1')$$

$$KV_c = aKV_{a1} + a^2 KV_{a2} + KV_{a0} = a(R_F + C_1 Z_1')$$

If delta-connected CT's are involved,

$$K\,(I_a - I_b) = (1 - a^2)\ C_1$$

$$K\,(I_b - I_c) = (a^2 - a)\ C_1$$

$$K(I_c - I_a) = (a - 1)\ C_1$$

The phase-to-phase voltages are:

$$KV_{ab} = K\,(V_a - V_b) = (1 - a^2)\ (R_F + C_1 Z_1')$$

$$KV_{bc} = K\,(V_b - V_c) = (a^2 - a)\ (R_F + C_1 Z_1')$$

$$KV_{ca} = K\,(V_c - V_a) = (a - 1)\ (R_F + C_1 Z_1')$$

PHASE-TO-PHASE SHORT CIRCUITS

Figure 5 shows the phase *a* phase-sequence networks for a phase-*b*-to-phase-*c* fault. By inspection, we can write:

$$I_1 = \frac{E_1}{Z_1 + Z_2 + R_F} = \frac{E_1}{2Z_1 + R_F}$$

(assuming $Z_2 = Z_1$). We shall let $E_1/(2Z_1 + R_F) = I/K$ for the same reason as for the three-phase short-circuit calculations, realizing that the values of K are not the same in both cases. It should also be realized that the values of R_F will probably be different in both cases.

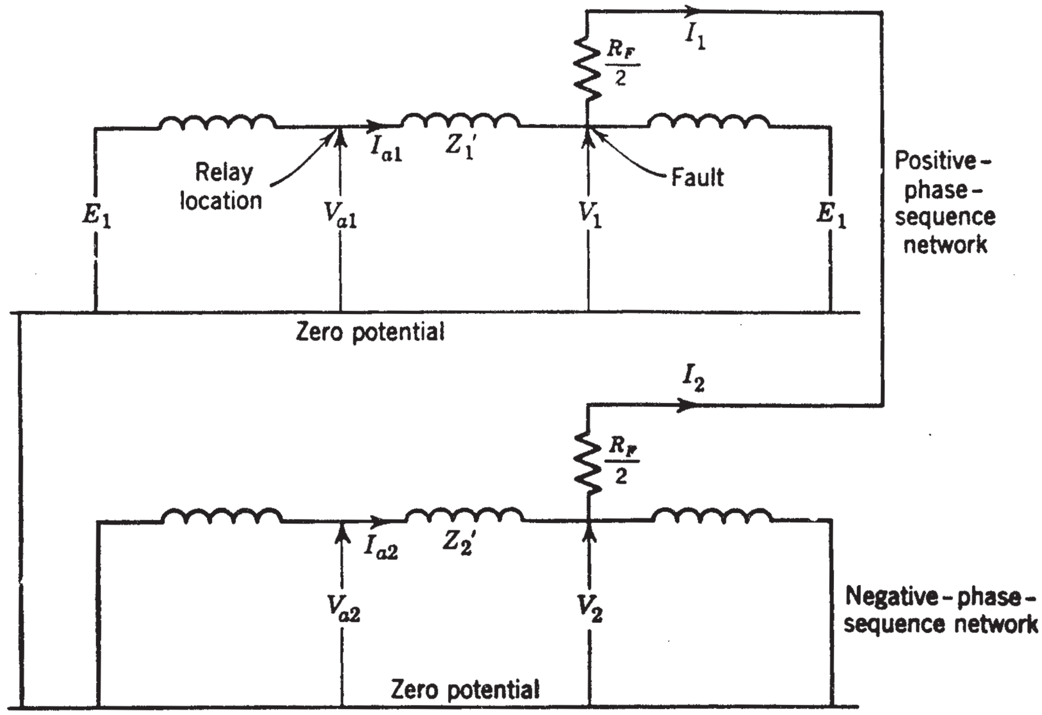

Fig. 5. Phase *a* phase-sequence networks for a phase-*b*-to-phase-*c* fault.

Here, R_F is the resistance between the faulted phases. Figure 5 shows $R_F/2$ in series with each network, in order to be consistent with certain references for this chapter (1,2,5). This is simply a caution not to attach too much significance to apparent differences in the R_F term. Continuing, we can write:

$$V_1 = (I_1 - I_2)\frac{R_F}{2} - I_2 Z_2$$

$$= I_1 R_F + I_1 Z_1 \text{ since } I_2 = -I_1 \text{ and } Z_2 = Z_1 \text{ (assumed)}$$

$$V_{a1} = V_1 + I_{a1} Z_1'$$

$$= I_1 (R_F + Z_1 + C_1 Z_1') \text{ since } I_{a1} = C_1 I_1 \text{ by definition}$$

$$= \frac{1}{K}(R_F + Z_1 + C_1 Z_1')$$

$$KV_{a1} = R_F + Z_1 + C_1 Z_1'$$

$$V_2 = -I_2 Z_2 = I_1 Z_1$$

$$V_{a2} = V_2 + I_{a2} Z_2'$$

But $Z_2' = Z_1'$ for transmission lines, and $I_{a2} = C_2 I_2$ by definition, and hence:

$$V_{a2} = I_1 Z_1 + C_2 I_2 Z_1'$$

We shall further assume that $C_2 I_2 = -C_1 I_1$, and hence:

$$V_{a2} = I_1 Z_1 - C_1 I_1 Z_1'$$

$$= I_1 (Z_1 - C_1 Z_1')$$

$$KV_{a2} = Z_1 - C_1 Z_1'$$

By definition, $I_{a1} = C_1 I_1$,

$$KI_{a1} = C_1$$

Since $I_{a2} = -I_{a1}$

$$KI_{a2} = -KI_{a1} = -C_1$$

Since there are no zero-phase-sequence quantities,

$$KV_{a0} = 0$$

$$KI_{a0} = 0$$

We could continue, as for the three-phase fault, and write the actual currents and voltages at the relay location, but the technique is the same as before. The final values are given in Tables 2 and 3. Also,

Table 2. Currents during Faults

Quantity at Relay Location	Value of Quantity for Various Types of Fault: Three-Phase	Phase b to Phase c	Phase a to Ground
KI_{a1}	C_1	C_1	C_1
KI_{a2}	0	$-C_1$	C_1
KI_{a0}	0	0	C_0
KI_a	C_1	0	$C_0 + 2C_1$
KI_b	a^2C_1	$(a^2 - a)C_1$	$C_0 - C_1$
KI_c	aC_1	$-(a^2 - a)C_1$	$C_0 - C_1$
$K(I_a - I_b)$	$(1 - a^2)C_1$	$-(a^2 - a)C_1$	$3C_1$
$K(I_b - I_c)$	$(a^2 - a)C_1$	$2(a^2 - a)C_1$	0
$K(I_c - I_a)$	$(a - 1)C_1$	$-(a^2 - a)C_1$	$-3C_1$
$K(I_a + I_b + I_c)$	0	0	$3C_0$
K	$\dfrac{Z_1 + R_F}{E_1}$	$\dfrac{2Z_1 + R_F}{E_1}$	$\dfrac{2Z_1 + Z_0 + 3R_F}{E_1}$

the calculations for a phase-*a*-to-ground fault will be omitted, but the values are given in the tables. The tables do not include the values for a phase-*b*-to-phase-*c*-to-ground fault because they are too complicated, and are not very significant. The purpose here is not to present all the data, but to show the technique involved in determining it, and it is felt that in this respect enough data will have been given. Complete data will be found in two different AIEE papers.[4,5]

Table 3. Voltages during Faults

Quantity at Relay Location	Value of Quantity for Various Types of Fault: Three-Phase	Phase b to Phase c	Phase a to Ground
KV_{a1}	$C_1 Z_1{}' + R_F$	$C_1 Z_1{}' + Z_1 + R_F$	$C_1 Z_1{}' + Z_1 + Z_0 + 3R_F$
KV_{a2}	0	$Z_1 - C_1 Z_1{}'$	$C_1 Z_1{}' - Z_1$
KV_{a0}	0	0	$C_0 Z_0{}' - Z_0$
KV_a	$C_1 Z_1{}' + R_F$	$2Z_1 + R_F$	$2C_1 Z_1{}' + C_0 Z_0{}' + 3R_F$
KV_b	$a^2(C_1 Z_1{}' + R_F)$	$(a^2 - a)C_1 Z_1{}' - Z_1 + a^2 R_F$	$-C_1 Z_1{}' + (a^2 - a)Z_1 + (a^2 - 1)Z_0 + C_0 Z_0{}' + 3a^2 R_F$
KV_c	$a(C_1 Z_1{}' + R_F)$	$(a - a^2)C_1 Z_1{}' - Z_1 + aR_F$	$-C_1 Z_1{}' + (a - a^2)Z_1 + (a - 1)Z_0 + C_0 Z_0{}' + 3aR_F$
$K(V_a - V_b)$	$(1 - a^2)(C_1 Z_1{}' + R_F)$	$(a - a^2)C_1 Z_1{}' + 3Z_1 + (1 - a^2)R_F$	$3C_1 Z_1{}' - (a^2 - a)Z_1 - (a^2 - 1)(Z_0 + 3R_F)$
$K(V_b - V_c)$	$(a^2 - a)(C_1 Z_1{}' + R_F)$	$2(a^2 - a)C_1 Z_1{}' + (a^2 - a)R_F$	$2(a^2 - a)Z_1 + (a^2 - a)(Z_0 + 3R_F)$
$K(V_c - V_a)$	$(a - 1)(C_1 Z_1{}' + R_F)$	$(a - a^2)C_1 Z_1{}' - 3Z_1 + (a - 1)R_F$	$-3C_1 Z_1{}' + (a - a^2)Z_1 + (a - 1)(Z_0 + 3R_F)$
$K(V_a + V_b + V_c)$	0	0	$3(C_0 Z_0{}' - Z_0)$
K	$\dfrac{Z_1 + R_F}{E_1}$	$\dfrac{2Z_1 + R_F}{E_1}$	$\dfrac{2Z_1 + Z_0 + 3R_F}{E_1}$

DISCUSSION OF ASSUMPTIONS

The error in assuming the positive- and negative-phase-sequence impedances to be equal depends on the operating speed of the relay involved and on the location of the fault and relay relative to a generator. All the error is in the assumed generator impedances. Whereas the negative-phase-sequence reactance of a generator is constant, the positive-phase-sequence reactance will grow larger from subtransient to synchronous within a very short time after a fault occurs. However, unless the fault is at the terminals of a generator, the constant impedance of transformers and lines between a generator and the fault will tend to lessen the effect of changes in the generator's positive-phase-sequence reactance.

The assumption that positive- and negative-phase-sequence impedances are equal is sufficiently accurate for analyzing the response of high-speed distance-type relay units. For the short time that it takes a high-speed relay to operate after a fault has occurred, the equivalent positive-phase-sequence reactance of a generator is nearly enough equal to the negative-phase-sequence reactance so that there is negligible over-all error in assuming them to be equal. Actually, it is usually the practice to use the rated-voltage direct-axis transient reactance for both.

When time-delay distance-type units are involved, such as for backup relaying, the assumption of equal positive- and negative-phase-sequence impedances could produce significant errors. This is especially true for a fault near a generator and with the relay located between the generator and the fault, in which event the relay's operation would be largely dependent on the generator characteristics alone. On the other hand, the "reach" of a back-up relay does not have to be as precise as that of high-speed primary relaying, which permits more error in its adjustment. Suffice it to say that this possibility of error, even with time-delay relays, is generally ignored for distance relaying except in very special cases.

Actually, this consideration of the possible error resulting from assuming equal positive- and negative-phase-sequence impedances is somewhat academic where distance relays are involved. Whatever error there may be will generally affect the response of only those relays on whose operation we do not ordinarily rely anyway. An exception to this is for faults on the other side of a wye-delta or delta-wye transformer, which we shall consider later.

Neglecting the effect of mutual induction, where mutual induction exists, would noticeably affect the response of only ground relays. It is the practice to ignore mutual induction when dealing with phase relays. The effect of mutual induction on the response of ground distance relays may have to be considered; this is treated in detail in Reference 3.

DETERMINATION OF DISTANCE-RELAY OPERATION FROM THE DATA OF TABLES 2 AND 3

Modern distance relays are single-phase types. The three relays used for phase-fault protection are supplied with the following combinations of current and voltage:

Current	Voltage
$I_a - I_b$	$V_{ab} = V_a - V_b$
$I_b - I_c$	$V_{bc} = V_b - V_c$
$I_c - I_a$	$V_{ca} = V_c - V_a$

These quantities are often called "delta currents" and "delta voltages." Each relay is intended to provide protection for faults involving the phases between which its voltage is obtained.

It was shown in Chapter 4 that all distance relays operate according to some function of the ratio of the voltage to the current supplied to the relay. We are now able to determine what the ratio of these quantities is for each phase relay for any type of fault, by using the data of Tables 2 and 3. These ratios are given in Table 4.

Table 4. Impedances "Seen" by Phase Distance Relays for Various Kinds of Short Circuits

	Value of Ratio for Various Types of Fault		
Ratio	3-Phase	Phase b to Phase c	Phase a to Ground
$\dfrac{K(V_a - V_b)}{K(I_a - I_b)}$	$Z_1' + \dfrac{R_F}{C_1}$	$Z_1' - j\sqrt{3}\,Z_{X1} - \dfrac{a}{C_1}R_F$	$Z_1' + j\dfrac{\sqrt{3}}{3}Z_{X1} + \dfrac{(1-a^2)(Z_0+3R_F)}{3C_1}$
$\dfrac{K(V_b - V_c)}{K(I_b - I_c)}$	$Z_1' + \dfrac{R_F}{C_1}$	$Z_1' + \dfrac{R_F}{2C_1}$	∞
$\dfrac{K(V_c - V_a)}{K(I_c - I_a)}$	$Z_1' + \dfrac{R_F}{C_1}$	$Z_1' + j\sqrt{3}\,Z_{X1} - \dfrac{a^2}{C_1}R_F$	$Z_1' - j\dfrac{\sqrt{3}}{3}Z_{X1} - \dfrac{(a-1)(Z_0+3R_F)}{3C_1}$

For a three-phase fault, all three relays "see" the positive-phase-sequence impedance of the circuit between the relays and the fault, plus a multiple of the arc resistance. This multiple depends on the fraction of the total fault current that flows at the relay location and is larger for smaller fractions.

For a phase-to-phase fault at a given location, the relay energized by the voltage between the faulted phases sees the same impedance as for three-phase faults at that location, except, possibly, for differences in the R_F term. As already noted, the value of R_F may be different for the two types of fault. The subject of fault resistance will be treated later in more detail when we consider the application of distance relays. The other two relays see other impedances.

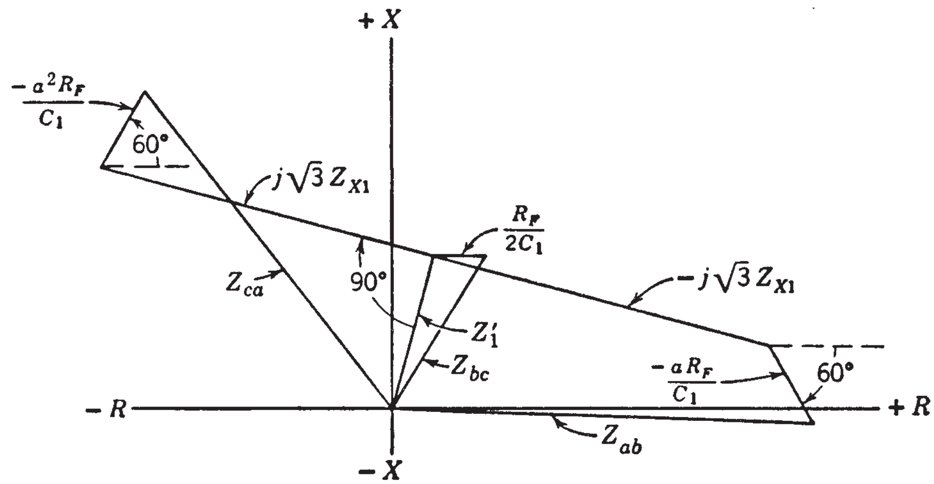

Fig. 6. Impedances seen by each of the three phase distance relays for a phase-*b*-to-phase-*c* fault.

These values of impedance seen by the three relays can be shown on an *R-X* diagram, as in Fig. 6. The terms Z_{bc}, Z_{ab}, and Z_{ca} identify the impedances seen by the relays obtaining voltage between phases *bc, ab,* and *ca,* respectively. If we were to follow rigorously the conventions already described for the *R-X* diagram, we should draw three separate diagrams, one each for the constructions for obtaining Z_{bc}, Z_{ab}, and Z_{ca}. This is because each one involves the ratio of different quantities. However, since the relay characteristics would be the same on all three diagrams, it is more convenient to put all impedance characteristics on the same diagram, and also it reveals certain interesting interrelations, as we shall see shortly.

For phase-*b*-to-phase-*c* faults, with or without arcs, and located anywhere on a line section from the relay location out to a certain distance, the heads of the three impedance radius vectors will lie on or within the boundaries of the shaded areas of Fig. 7. These areas would be generated if we were to let Z_1' and R_F of Fig. 6 increase from zero to the value shown.

To use the data shown by Fig. 7, it is only necessary to superimpose the characteristic of any distance relay using one of the combinations of delta current and voltage in order to determine its operating tendencies. This has been done on Fig. 7 for an impedance-type distance relay adjusted to operate for all faults having any impedance within the shaded area Z_{bc}. Had we shown the three fault areas Z_{ab}, Z_{bc}, and Z_{ca} on three different *R-X* diagrams, the relay characteristic would still have looked the same on all three diagrams since

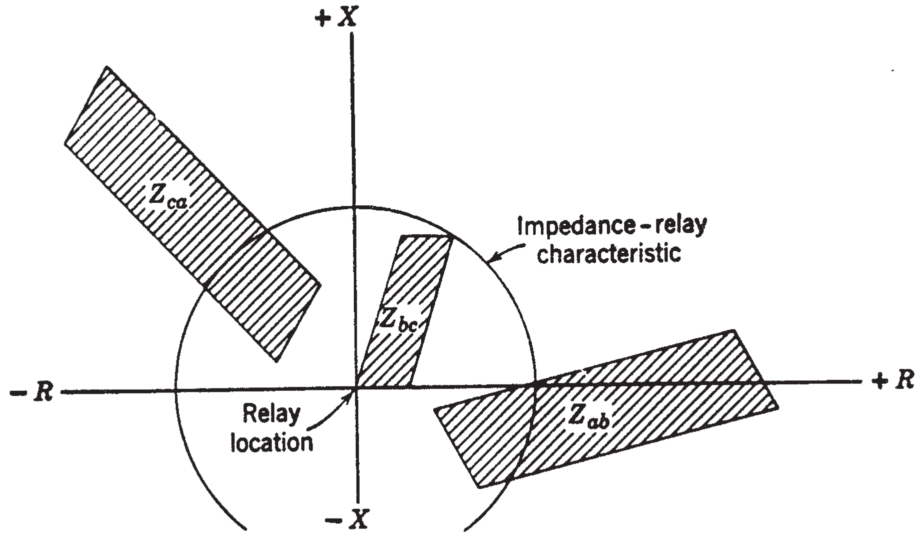

Fig. 7. Impedance areas seen by the three phase distance relays for various locations of a phase-*b*-to-phase-*c* fault with and without an arc.

the practice is to adjust all three relays alike. Therefore, the relay characteristic of Fig. 7 may be thought of as that of any one of the three relays. For any portion of shaded area lying inside the relay characteristic, it is thereby indicated that for certain locations of the phase-*b*-to-phase-*c* fault, the relay represented by that area will operate.

For the adjustment of Fig. 7, all three relays will operate for nearby faults, represented by certain values of Z_{ca} and Z_{ab}, where the shaded areas fall within the operating characteristic of the impedance relay. Such operation is not objectionable, but the target indications might lead one to conclude that the fault was three-phase instead of phase-to-phase.

We may generalize the picture of Fig. 7, and think of the Z_{bc} area as representing the appearance of a phase-to-phase fault to the distance relay that is supposed to operate for that fault. Then, the Z_{ca} area shows the appearance of the fault to the relay using the voltage lagging the faulted phase voltage (sometimes called the "lagging" relay); and the Z_{ab} area shows the appearance of the fault to the relay using the voltage leading the faulted phase voltage (sometimes called the "leading" relay).

We can construct the diagram of Fig. 6 graphically, as in Fig. 8, neglecting the effect of arc resistance. Draw the line *OF* equal to $Z_1{}'$ of Fig. 6. Extend *OF* to *A*, making *FA* equal to Z_{X1},

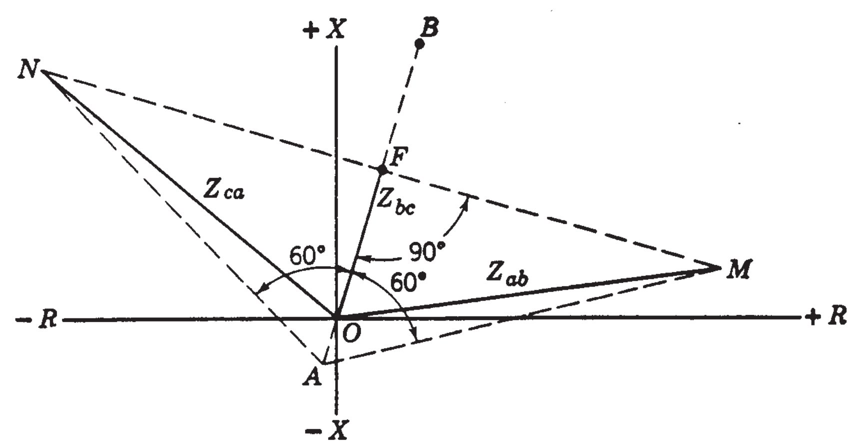

Fig. 8. Graphical construction of Fig. 6, neglecting the effect of arc resistance.

the positive-phase-sequence impedance from the fault to the end of the system back of the relay location. Actually, *FA* is Z_{X2}, but we are assuming the positive- and negative-phase-sequence impedances to be equal. Draw a line through *F* perpendicular to *AF*. From *A*, draw lines at 60° to *AF* until they intersect the perpendicular to *AF* at *M* and *N*. Then:

$$OF = Z_{bc}$$

$$OM = Z_{ab}$$

$$ON = Z_{ca}$$

The proof that this construction is valid is that the tangent of the angle *FAM* is equal to:

$$\frac{\sqrt{3}\ Z_{X1}}{Z_{X1}} = \sqrt{3}$$

which is the tangent of 60°. The construction for showing the effect of fault resistance can be added to Fig. 8 by the method used for Fig. 6.

It should be noted that the line segments *FO* and *OA* would not lie on the same straight line as in Fig. 8 if their *X/R* ratios were different. Then a straight line *FA* that did not go through the origin would be drawn, and the rest of the construction would be based on it, the perpendicular to it being drawn through *F*, and the lines *AM* and *AN* being drawn at 60° to it.

The appearance of a phase-*a*-to-ground fault to phase distance relays is shown in Fig. 9. Except for the last term in Table 4, this diagram can be constructed graphically by drawing the two construction lines at 30° to *FA*.

Similar constructions can be made for distance relays used for ground-fault protection. However, these constructions will not be described here. A paper containing a complete treatment of this subject is listed in the Bibliography.[5]

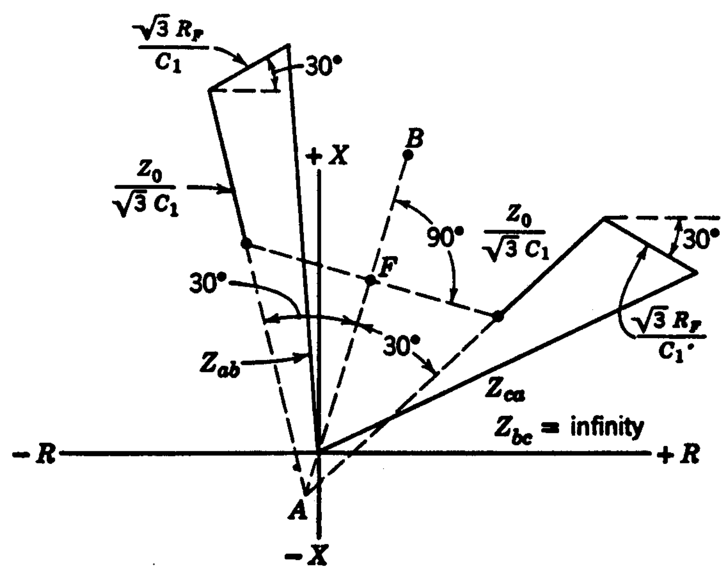

Fig. 9. Appearance of a phase-*a*-to-ground fault to phase distance relays.

The foregoing constructions, showing what different kinds of faults look like to distance relays, are of more academic than practical value. In other words, although these constructions are extremely helpful for understanding how distance relays respond to different kinds of faults, one seldom has to make such constructions to apply distance relays. It is usually only necessary to locate the point representing the appearance of a fault to the one relay that should operate for the fault. In other words, only the positive-phase-sequence impedance between the relay and the fault is located. The information gained from such constructions explains why relay target indications cannot always be relied on for determining what kind of fault occurred; in other words, three targets (apparently indicating a three-fault phase fault) might show for a nearby phase-to-phase fault. Or a phase relay might show a target for a nearby single-phase-to-ground fault, etc. The construction has also been useful for explaining a tendency of certain ground relays to "overreach" for phase faults;[5] because of this tendency it is customary to provide means for blocking tripping by ground distance relays when a fault involves two or more phases, or at least to block tripping by the ground relays that can overreach.

The principal use of the foregoing type of construction for practical application purposes is when one must know how distance relays will respond to faults on the other side of a power transformer. This will now be considered.

EFFECT OF A WYE-DELTA OR A DELTA-WYE POWER TRANSFORMER BETWEEN DISTANCE RELAYS AND A FAULT

For other than three-phase faults, the presence of a wye-delta or delta-wye transformer between a distance relay and a fault changes the complexion of the fault as viewed from the distance-relay location,[6] because of the phase shift and the recombination of the currents and voltages from one side to the other of the power transformer. In passing through the transformer from the fault to the relay location, the positive-phase-sequence currents and voltages of the corresponding phases are shifted 30° in one direction, and the negative-phase-sequence quantities are shifted 30° in the other direction. The zero-phase-sequence quantities are not transmitted through such a power transformer. The 30° shift described here is not at variance with the 90° shift described in some textbooks on symmetrical components. The 90° shift is a simpler mathematical manipulation, but it does not apply to what is usually considered the *corresponding* phase quantities.

In terms of only the magnitude of per unit quantities in an equivalent system diagram, the only effect of the presence of a power transformer is its impedance in the phase-sequence circuits. But, to combine the per unit phase-sequence quantities and convert them to volts and amperes at the relay location, one must first shift the per unit quantities by the proper phase angle from their positions on the fault side of the power transformer. If the power transformer has the standard connections described in Chapter 8 whereby the high-voltage phase currents lead the corresponding low-voltage phase currents by 30° under balanced three-phase conditions the positive-phase-sequence currents and voltages on the HV side lead the corresponding positive-phase-sequence components on the LV side by 30°. (Under balanced three-phase conditions, only positive-phase-sequence quantities exist, and the vector diagram for this condition is a positive-phase-sequence diagram; this is a good way to determine the direction and amount of shift for any connection.) The negative-phase-sequence quantities on the HV side lag the corresponding LV quantities by 30°, or, in other words, they are shifted by the same amount as the positive-phase-sequence quantities, but in the opposite direction.

For three-phase faults where there are only positive-phase-sequence currents and voltages, the fact that these quantities are shifted 30° in going through the transformer may be ignored because all of them are shifted in the same direction. On the *R-X* diagram, a three-phase fault on the other side of a power transformer is represented merely by adding to the positive-phase-sequence impedance between the relay and the transformer the positive-phase-sequence impedance of the transformer and of the line between the transformer and the fault. If the impedances are in ohms, it is only necessary to be sure that the impedances of the transformer and the line between it and the fault are expressed in terms of the rated voltage of the transformer on the relay side; if the impedances are in percent or per unit, each impedance should be based on the base voltage of the portion of the circuit in which it exists, exactly the same as for any short-circuit study. It is probably evident that all three relays will operate alike for a three-phase fault.

The net effect of the shift in the positive- and negative-phase-sequence components on the impedance appearing to a distance relay on the HV side for a fault on the LV side is compared in Table 5 with data from Table 4 for a phase-*b*-to-phase-*c* fault. For the purposes of comparison, the same impedance values between the relay and the fault are assumed for both fault locations, and the effect of fault resistance is neglected.

Table 5. Comparison of Appearance of Phase-*b*-to-Phase-*c* Faults on Either Side of a Wye-Delta or Delta-Wye Power Transformer to Distance Relays on the HV Side

	HV Fault (From Table 4)	LV Fault
Z_{ab}	$Z_1{}' - j\sqrt{3}\, Z_{X1}$	$Z_1{}' - j\frac{\sqrt{3}}{3} Z_{X1}$
Z_{bc}	$Z_1{}'$	$Z_1{}' - j\frac{\sqrt{3}}{3} Z_{X1}$
Z_{ca}	$Z_1{}' - j\sqrt{3}\, Z_{X1}$	∞

The effect of the *LV* fault on the graphical construction for showing these impedances on an *R-X* diagram is to shift the construction lines *AM*, *AN*, and *AF* by 30° in the counter clockwise direction from their positions in Fig. 8 to the new positions *AM'*, *AN'*, and *AF'* as illustrated in Fig. 10. In other words, $Z_{ab} = OM'$, $Z_{bc} = OF'$, and Z_{ca} = infinity because the construction line is parallel to the line *MN*. Table 5 and Fig. 10 apply whether the power transformer is connected wye-delta or delta-wye, so long as it has the standard connections described in Chapter 8. It will be noted that the construction of Fig. 10 also uses as a starting point the line *OF* which represents the positive-phase-sequence impedance for a three-phase fault at the fault location, including the transformer impedance.

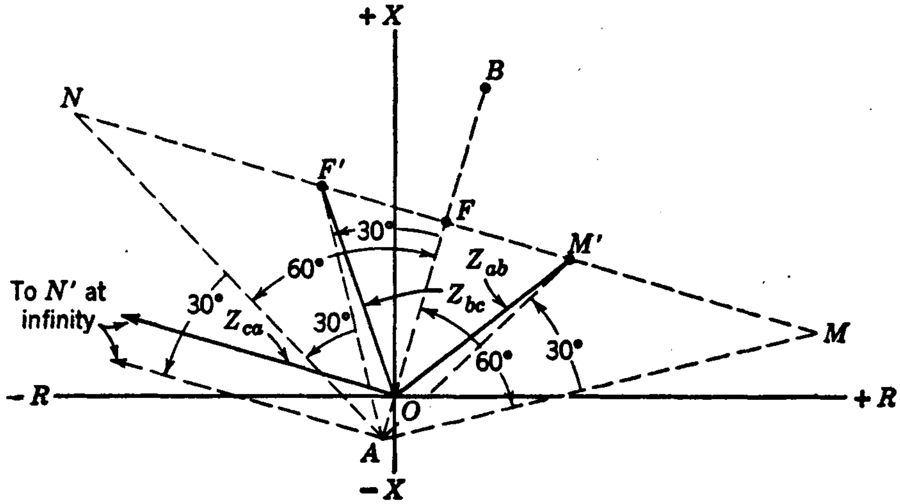

Fig. 10. Appearance to phase distance relays of a phase-*b*-to-phase-*c* fault on the low-voltage side of a wye-delta or delta-wye power transformer.

If the relay is on the LV side of the power transformer, and the phase-*b*-to-phase-*c* fault is on the HV side, the construction of Fig. 11 applies. It will be noted that here the construction lines are shifted 30° clockwise from their positions for the fault on the same side as the relay.

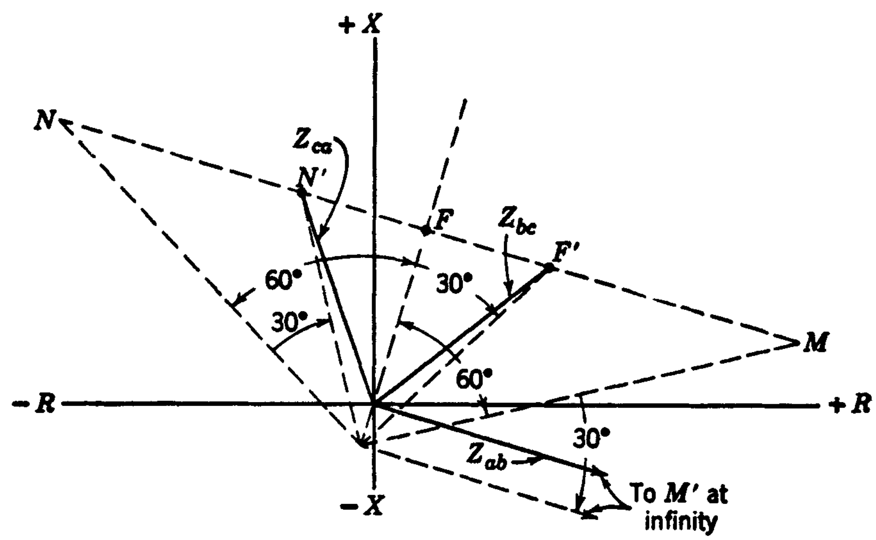

Fig. 11. Appearance, to phase distance relays on the LV side, of a phase-*b*-to-phase-*c* fault on the HV side of a wye-delta or delta-wye power transformer.

Comparing the foregoing *R-X* diagrams, we note that as we move the fault from the relay side of the power transformer to the other side, the construction lines shift 30° in the same direction that the negative-phase-sequence components shift in going through the transformer from the relay to the other side. This assumes that the transformer has the standard connections.

Space does not permit the consideration of the appearance to phase distance relays of single-phase-to-ground faults on the other side of a transformer, or of the appearance to ground distance relays of various types of faults on the other side of a transformer. Suffice it to say, a single-phase-to-ground fault on one side looks like a phase-to-phase fault on the other side, and vice versa, when wye-delta or delta-wye power transformers are involved.

The significant information that we get from studies of this kind may be summarized as follows:

1. If we want to be sure that any kind of distance relay on one side will not operate for any kind of fault on the other side, we must be sure that it will not operate for a three-phase fault.

2. If we want to be sure that a phase distance relay on one side will operate for any kind of fault on the other side, we must be sure that it will operate for a phase-to-phase fault.

3. If we want to be sure that a ground distance relay on one side will operate for any kind of fault on the other side, we must be sure that it will operate for a single-phase-to-ground fault. This assumes that the system neutral is grounded in such a way that a single-phase-to-ground fault at the location under consideration will cause short-circuit current to flow at the relay location.

After one has tried to adjust distance relays to provide back-up protection for certain faults on the other side of a transformer bank, it will become evident that it would be more practical either to use backup units connected to measure distance correctly for such faults or to use the “reversed-third-zone” principle which will be described in Chapter 14. The connection of back-up units to measure distance correctly is the same principle as that when low-tension current and voltage are used, except that the high-tension quantities must be so combined as to duplicate the low-tension quantities.

POWER SWINGS AND LOSS OF SYNCHRONISM

Power swings are surges of power such as those after the removal of a short circuit, or those resulting from connecting a generator to its system at an instant when the two are out of phase. The characteristic of a power swing is the same as the early stages of loss of synchronism, and hence the loss-of-synchronism characteristic can describe both phenomena.

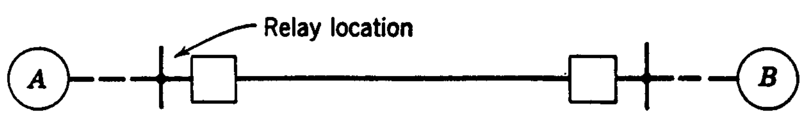

Fig. 12. One-line diagram of a system, illustrating loss-of-synchronism characteristics.

Consider the one-line diagram of Fig. 12 where a section of transmission line is shown with generating sources beyond either end of the line section. As for short-circuit studies, it is the practice, when possible, to represent the system by its two-generator equivalent.

The location of a relay is shown whose response to loss of synchronism between the two generating sources is to be studied. Each generating source may be either an actual generator or an equivalent generator representing a group of generators that remain in synchronism. (If generators within the same group lose synchronism with each other, this simple approach to the problem cannot be used, and a network-analyzer study may be required to provide the desired data.) The effects of shunt capacitance and of shunt loads are neglected.

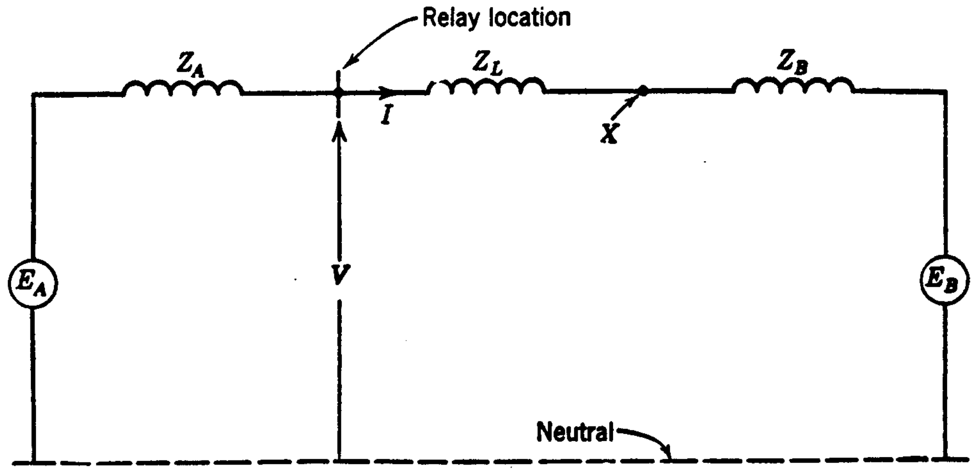

Fig. 13. System constants and relay current and voltage for Fig. 12.

Figure 13 indicates the pertinent phase-to-neutral (positive-phase-sequence) impedances and the generated voltages of the system of Fig. 12, and also the phase current and phase-to-neutral voltage at the relay location. An equivalent generator reactance of 90% of the direct-axis rated-current transient reactance most nearly represents a generator during the early stages of a power swing, and should be used for calculating the impedances in such an equivalent circuit.[7] In practice the generator reactance and the generated voltage are assumed to remain constant.

We can derive the relay quantities as follows:

$$I = \frac{E_A - E_B}{Z_A + Z_L + Z_B}$$

$$V = E_A - IZ_A = E_A - \frac{(E_A - E_B)\ Z_A}{Z_A + Z_L + Z_B}$$

$$\frac{V}{I} = Z = \frac{E_A}{E_A - E_B}\ (Z_A + Z_L + Z_B) - Z_A$$

If we let E_B be the reference, and let E_A advance in phase ahead of E_B by the angle θ, and if we let the magnitude of E_A be equal to nE_B, where n is a scalar, then

$$\frac{E_A}{E_A - E_B} = \frac{n\ (\cos\theta + j\sin\theta)}{n\ (\cos\theta + j\sin\theta) - 1}$$

This equation will resolve into the form:

$$\frac{E_A}{E_A - E_B} = \frac{n\ [(n\cos\theta) - j\sin\theta]}{(n - \cos\theta)^2 + \sin^2\theta}$$

If we take the special case where $n = 1$, the equation becomes:

$$\frac{E_A}{E_A - E_B} = \frac{1}{2}\left(1 - j \cot \frac{\theta}{2}\right)$$

Therefore, Z becomes:

$$Z = \frac{Z_A + Z_L + Z_B}{2}\left(1 - j \cot \frac{\theta}{2}\right) - Z_A$$

This value of Z is shown on the R-X diagram of Fig. 14 for a particular value of θ less than 180°. Point P is thereby seen to be a point on the loss-of-synchronism characteristic. Further thought will reveal that all other points on the loss-of-synchronism characteristic will lie on the dashed line through P. This line is the perpendicular bisector of the straight line connecting A and B.

The loss-of-synchronism characteristic has been expressed in terms of the ratio of a phase-to-neutral voltage to the corresponding phase current. Under balanced three-phase conditions that exist during loss of synchronism, this ratio is exactly the same as the ratio of delta voltage to delta current that was used earlier in this chapter for describing the appearance of short circuits to phase distance relays. Therefore, it is permissible to superimpose such loss-of-synchronism characteristics, short-circuit characteristics, and distance-relay characteristics on the same R-X diagram. For example, a three-phase fault (X of Fig. 13) at the far end of the line section from the relay location appears to distance relays as a point at the end of Z_L as shown on Fig. 14.

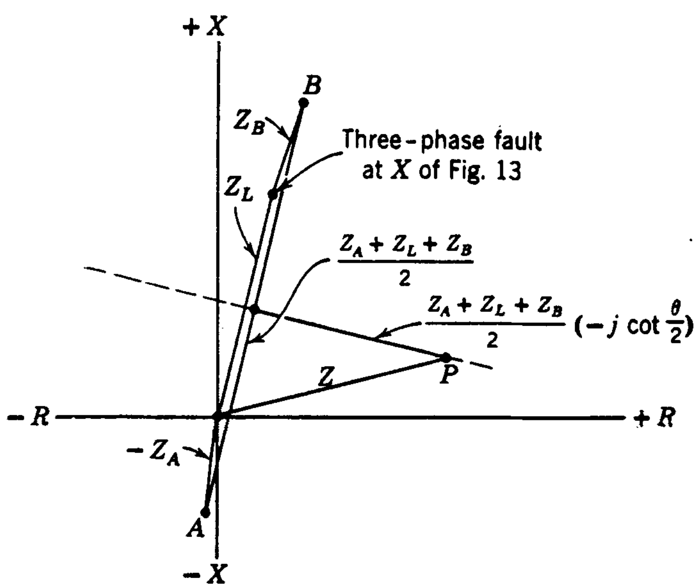

Fig. 14. Construction for locating a point on the loss-of-synchronism characteristic.

The foregoing observation will lead to the further observation that the point where the loss-of-synchronism characteristic intersects the impedance Z_L would also represent a three-phase fault at that point. In other words, at one instant during loss of synchronism, the conditions are exactly the same as for a three-phase fault at a point approximately midway electrically between the ends of the system. This point is called the "electrical center" or the "impedance center" of the system. This point would be exactly midway if the various impedances had the same *X/R* ratio, or, in other words, if all of them were in line on the diagram. The point where the loss-of-synchronism characteristic intersects the total-impedance line *AB* is reached when generator *A* has advanced to 180° leading generator *B*.

As shown in Fig. 15, the location of *P* for any angle θ between the generators can be found graphically by drawing a straight line from either end of the total-impedance line *AB* at the angle $\left(90 - \frac{\theta}{2}\right)$ to *AB*.

The point *P* is the intersection of this straight line with the loss-of-synchronism characteristic, which is the perpendicular bisector of the line *AB*. When θ is 90°, *P* lies on the circle whose diameter is the total-impedance line *AB*. This fact is useful to remember because it provides a simple method to locate a point corresponding to about the maximum load transfer.

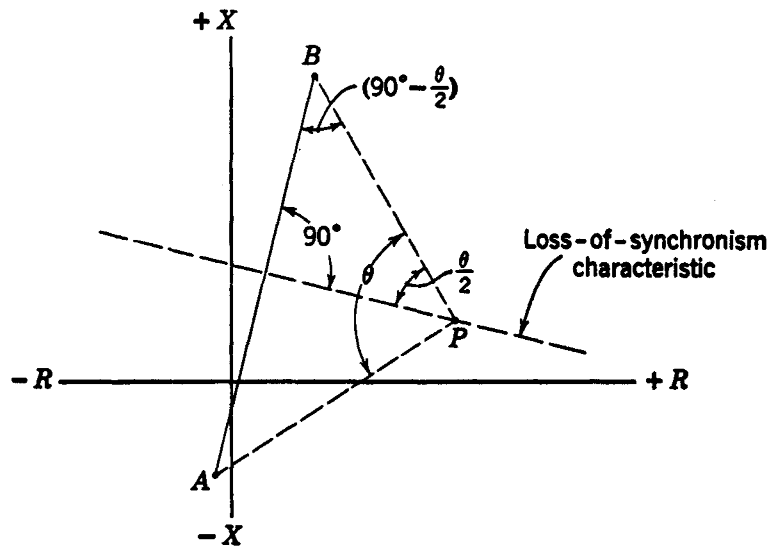

Fig. 15. Locating any point on the loss-of-synchronism characteristic corresponding to any value of θ.

The preceding development of the loss-of-synchronism characteristic was for the special case of $n = 1$. For most purposes, the characteristic resulting from this assumption is all that one needs to know in order to understand distance-type-relay response to loss of synchronism. However, without going into too much detail, let us at least obtain a qualitative picture of the general case where n is greater or less than unity.

All loss-of-synchronism characteristics are circles with their centers on extensions of the total-impedance line *AB* of Fig. 15. The characteristic when n = 1 is a circle of infinite radius. Any of these characteristics could be derived by successive calculations, if we assumed a value for n and then let θ vary from 0 to 360° in the general formula:

$$Z = (Z_A + Z_L + Z_B)n \frac{(n - \cos\theta) - j\sin\theta}{(n - \cos\theta)^2 + \sin^2\theta} - Z_A$$

Or one might manipulate the formula mathematically so as to get expressions for the diameter and the location of the center of the circle for any value of n. Figure 16 shows three loss-of-synchronism characteristics for $n > 1$, $n = 1$, and $n < 1$. The total-system-impedance line is again shown as AB.

The dashed circle through A, B, P', P, and P'' is an interesting aspect because all points on this circle to the right of the line AB, such as P', P, and P'' are for the same angle θ by which generator A has advanced ahead of generator B. This might be expected in view of the fact that the angle between a pair of lines drawn from any point on this part of the circle to A and B is equal to the angle between another pair of lines drawn from any other point on this part of the circle to A and B.

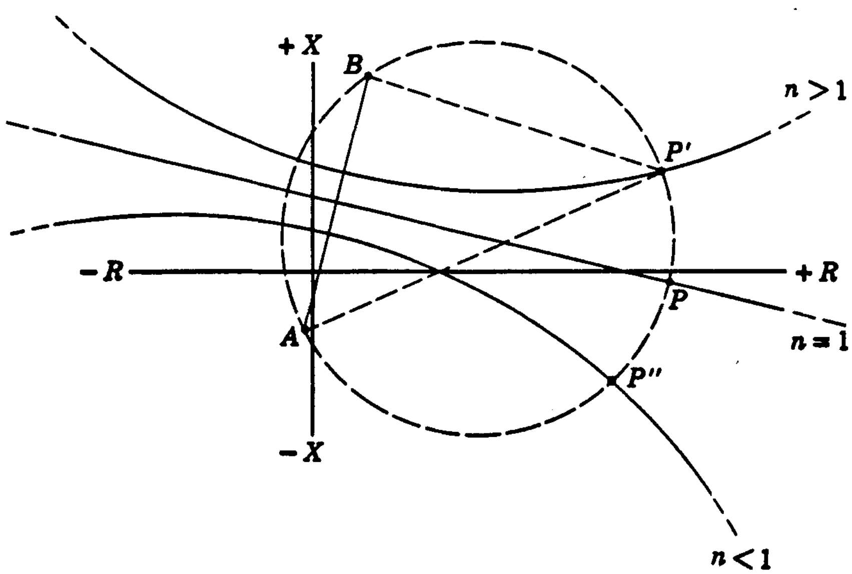

Fig. 16. General loss-of-synchronism characteristics.

Another interesting aspect of the diagram of Fig. 16 is that the ratio of the lengths of a pair of straight lines drawn from any point on the right-hand part of the circle to A and B will be equal to n. In other words, $P'A/P'B = n$. This suggests a simple method by which the loss-of-synchronism characteristic can be constructed graphically for any value of n. With a compass, one can easily locate three such points, all of which satisfy this same relation; having three points, one can then draw the circle.

This also suggests how to derive mathematical expressions for the radius of the circle and the location of its center. It is only necessary to assume the two possible locations of P' on the line AB and its extension, as shown on Fig. 17, and to obtain the characteristics of the circle that satisfy the relation $P'A/P'B = n$ for both locations.

According to this suggestion, it can be shown that, if we let Z_T be the total system impedance, then, for $n > 1$:

$$\text{Distance from B to center of circle} = \frac{Z_T}{n^2 - 1}$$

$$\text{Radius of the circle} = \frac{nZ_T}{n^2 - 1}$$

These are illustrated in Fig. 17.

The circles for $n < 1$ are symmetrical to those for $n > 1$, but with their centers beyond A; the same formulae can be used if $1/n$ is inserted in place of n.

The construction of the loss-of-synchronism characteristic is more complicated if the effects of shunt capacitance and loads are taken into account. Also, the presence of a fault during a power swing or loss of synchronism further complicates the problem. However, seldom if ever will one need any more information than has been given here. The entire subject is treated most comprehensively in references 5 and 8 of the Bibliography.

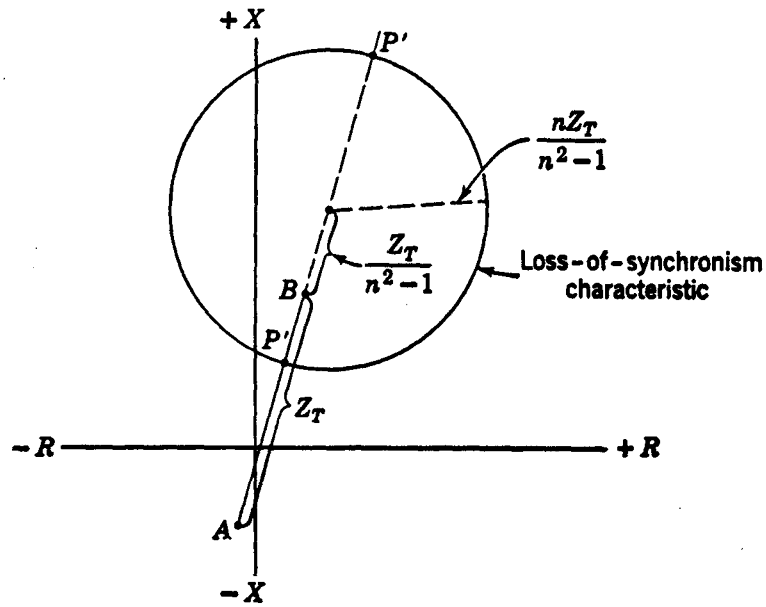

Fig. 17. Graphical construction of loss-of-synchronism characteristic.

EFFECT ON DISTANCE RELAYS OF POWER SWINGS OR LOSS OF SYNCHRONISM

To determine the response of any distance relay, it is only necessary to superimpose its operating characteristic on the R-X diagram of the loss-of-synchronism characteristic. This will show immediately whether any portion of the loss-of-synchronism characteristic enters the relay's operating region.

The fact that the loss-of-synchronism characteristic passes through or near the point representing a three-phase short circuit at the electrical center of the system indicates that any distance relay whose range of operation includes this point will have an operating tendency. Whether the relay will actually complete its operation and trip its breaker depends on the operating speed of the relay and the length of time during which the loss-of-synchronism conditions will produce an operating tendency. Only the back-up-time step of a distance relay is likely to involve enough time delay to avoid such tripping. For a given rate of slip S in cycles per second, one can determine how long the operating tendency will last. In Fig. 18, $(\theta' - \theta)$ gives the angular change of the slipping generator in degrees relative to the other generator. If we assume a constant rate of slip, the time during which

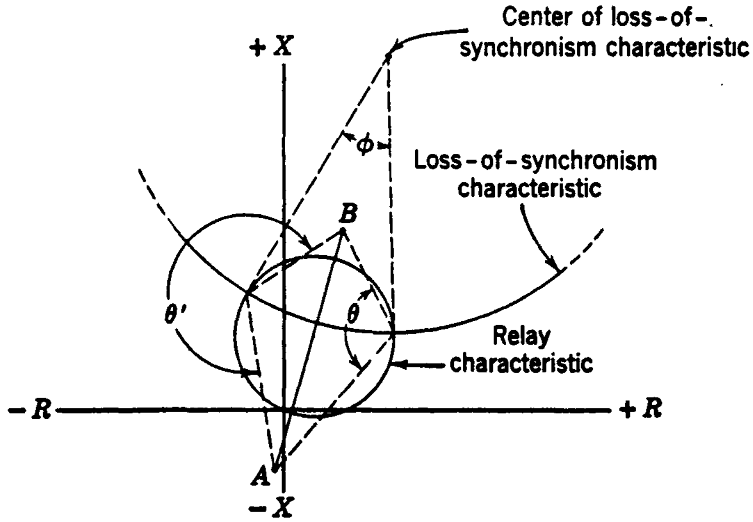

Fig. 18. Determination of relay operating tendency during loss of synchronism.

an operating tendency will last is $t = (\theta' - \theta)/360S$ seconds. Note that the angular change is not given by the angle ϕ of Fig. 18; once around the loss-of-synchronism circle is one slip cycle, but the rate of movement around the circle is not constant for a constant rate of slip.

The effect of changes in system configuration or generating capacities on the location of the loss-of-synchronism characteristic should be taken into account in determining the operating tendencies of distance relays. Such changes will shift the position of the electrical center with respect to adjoining line sections, and hence may at one time put the electrical center within reach of relays at one location, and at another time put it within reach of relays at another location.

This subject will be treated in more detail when we consider the application of distance relays to transmission-line protection.

RESPONSE OF POLYPHASE DIRECTIONAL RELAYS TO POSITIVE- AND NEGATIVE-PHASE-SEQUENCE VOLT-AMPERES

It has been shown[9] that the torque of polyphase directional relays can be expressed in terms of positive- and negative-phase-sequence volt-amperes. The proof of this fact involves the demonstration that only currents and voltages of the same phase sequence produce net torque in a polyphase directional relay. Moreover, when delta voltages or delta currents energize a polyphase directional relay, zero-phase-sequence currents produce no net torque because there are no zero-phase-sequence components in the delta quantities.

Making use of the foregoing facts, let us examine briefly the information that can be derived regarding the torques produced by the various conventional connections described in Chapter 3. Consider first the quadrature connection.

Figure 19 shows the corresponding positive- and negative-phase-sequence currents and voltages that produce net torque in one element of a polyphase directional relay; each of the other two elements will produce the same net torque since the positive- and negative-phase-sequence currents and voltages are balanced three phase. Additional torques are produced in each of the three elements by currents and voltages of opposite phase sequence, and these torques are not necessarily equal in each element, but they add to zero in the three elements, and hence can be neglected. (If single-phase directional relays were involved, these additional torques would have to be considered.)

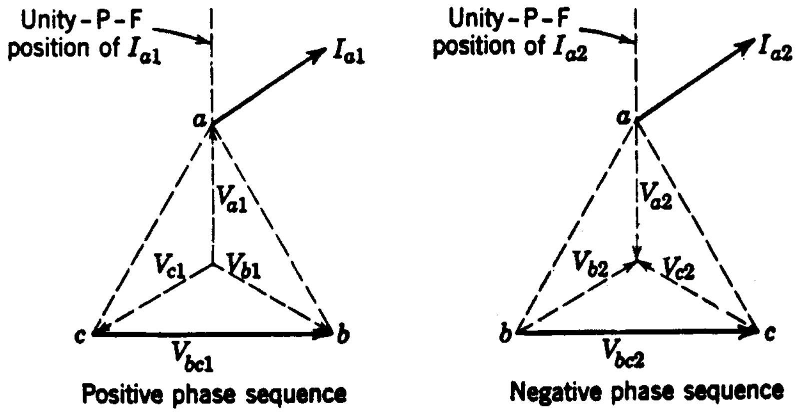

Fig. 19. Corresponding positive- and negative-phase-sequence torque-producing quantities for the quadrature connection.

Before proceeding to develop the torque relations, let us note that in Fig. 19 the arrows are on opposite ends of the phase-to-neutral voltages for the two phase sequences. The reason for this is that negative-phase-sequence voltages are voltage drops, whereas positive-phase-sequence voltages are voltage rises minus voltage drops; therefore, if corresponding positive- and negative-phase-sequence currents are in phase, their voltages are 180° out of phase, if we assume positive-, negative-, and zero-phase-sequence impedances to have the same X/R ratio. Another observation is that I_{a2} being shown in phase with I_{a1} has no significance; the important fact is that I_{a2} is assumed to lag the indicated unity-power-factor position by the same angle as I_{a1}. (The justification for this assumption is given later.)

If I_{a1} is β degrees from its maximum-torque position, the total torque is:

$$T \propto (V_{bc1}I_{a1} + V_{bc2}I_{a2}) \cos \beta$$

(Note that the voltage and current values in this and in the following torque equations are positive rms magnitudes, and should not be expressed as complex quantities. Note also that β may be positive or negative because the current may be either leading or lagging its maximum-torque position.) We can simplify the foregoing relation to:

$$T \propto (V_1I_1 + V_2I_2) \cos \beta$$

where the subscripts 1 and 2 denote positive- and negative-phase-sequence components, respectively, of the quantities supplied to the relay.

The vector diagrams for the 60° connection are shown in Fig. 20. The torque relations for Fig. 20 are:

$$T \propto V_1I_1 \cos \beta + V_2I_2 \cos (60° + \beta)$$

Likewise, the torque for the 30° connection is:

$$T \propto V_1I_1 \cos \beta + V_2I_2 \cos (120° + \beta)$$

From the foregoing relations, the difference in response for the various connections depends on the relative magnitude of the negative- voltage-current product to the positive-phase-sequence product. If there are no negative-phase-sequence quantities, the response is the same for any connection. The negative-phase-sequence product can vary from zero to a value equal to the positive-phase-sequence product. Equality of the products occurs for a phase-to-phase fault at the relay location with no fault resistance. For such a fault, our assumption of the same X/R ratio is most legitimate since we are dealing only with the positive- and negative-phase-sequence networks where this ratio is practically the same. Figure 21 shows the torque components and total torques versus, β for this extreme condition where both products are equal. The so-called "zero-degree" connection will be discussed later.

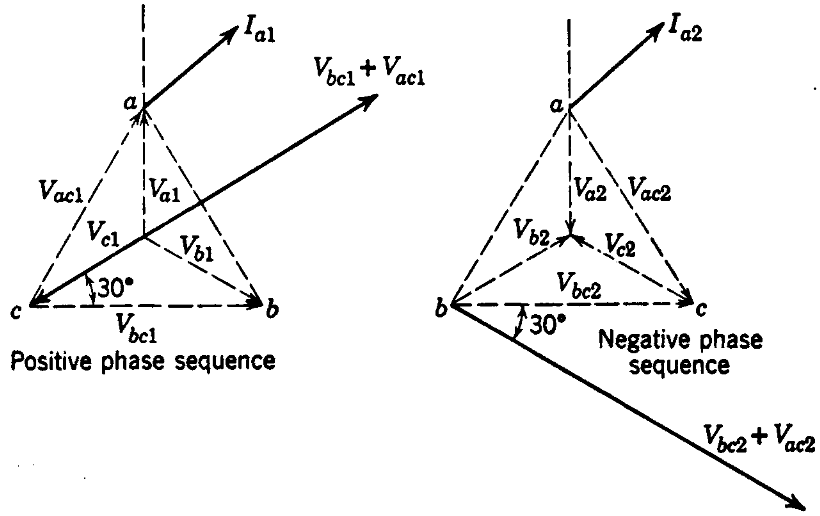

Fig. 20. Corresponding torque-producing quantities for the 60° connection.

For the limiting conditions of Fig. 21, if we assume that we want to develop positive net torque over the largest possible range of β within ±90°, the 90° connection is seen to be the best, and the 30° connection is the poorest. Remember that we may have balanced three-phase conditions alone, for which the torque is simply $V_1I_1 \cos \beta$, or we may have unbalanced conditions for which the total torque will be produced. Therefore, under the limiting conditions of a phase-to-phase fault at the relay location, and with no fault resistance, the three connections will produce positive torque over the total range of β shown in the accompanying table.

Connection	Range of β
Quadrature	180°
60°	150°
30°	120°

The ranges for the 60° and 30° connections will increase and approach 180° as the negative-phase-sequence product becomes smaller and smaller relative to the positive-phase-sequence product, or, in other words, as the fault is farther and farther away from the relay location. The fact that under short-circuit conditions the current does not range over the maximum theoretical angular limits (current limited only by resistance or only by inductive reactance) makes all three connections usable, but the quadrature connection provides the largest margin of safety for correct operation. (The possibility that capacity reactance may limit the current is not considered here, but it is an important factor when series capacitors are used in lines.)

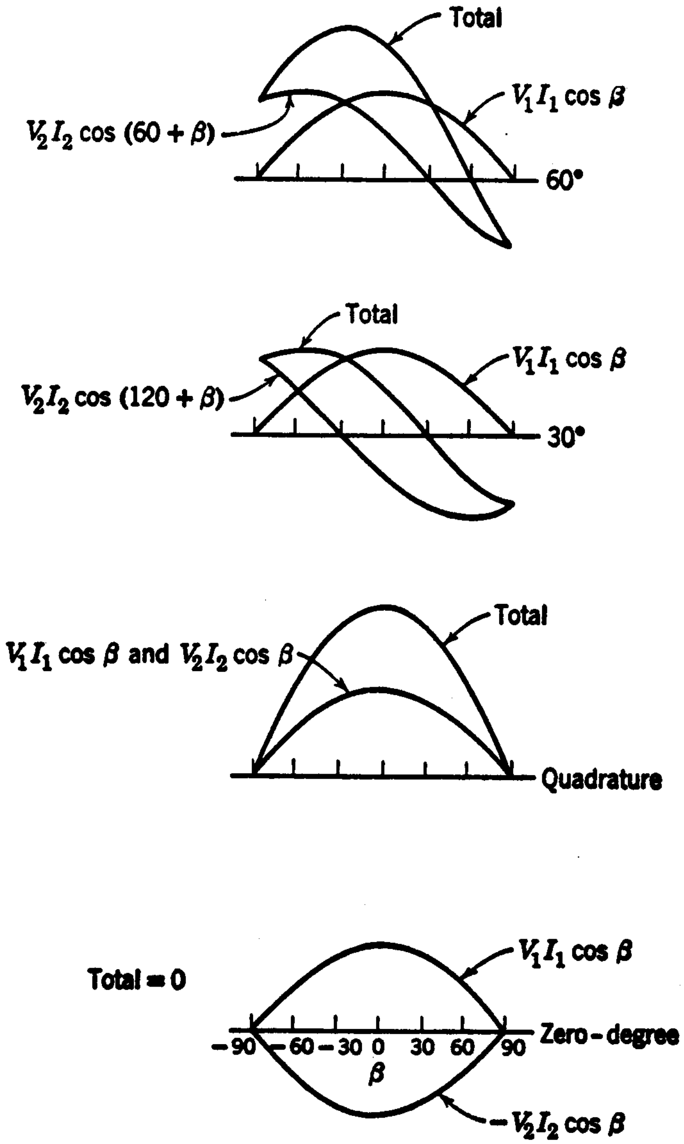

Fig. 21. Component and total torques of polyphase directional relays for conventional connections.

The zero-degree connection is that for which the current and voltage supplied to each relay element are in phase under balanced three-phase unity-power-factor conditions, as, for example, $I_a - I_b$ and V_{ab}. If there are no zero-phase-sequence components present, I_a and V_a could also be used. The equation for the torque produced with this connection is:

$$T \,\alpha\, V_1I_1 \cos \beta + V_2I_2 \cos (180° + \beta)$$

$$\alpha\, (V_1I_1 - V_2I_2) \cos \beta$$

The zero-degree connection will produce positive net torque over the same range of β as the quadrature connection, but the magnitude of the net torque over most of the range is less than for the other connections. For the limiting condition assumed for Fig. 21, the net torque is zero. The zero-degree connection is mentioned here to show that, by adding or subtracting the torques of a 90° polyphase relay and a zero-degree polyphase relay, operation can be obtained from either positive- or negative-phase-sequence volt-amperes. However, a most precise balance of the quantities must be effected if the net torque is to be truly representative of the desired quantity.

It follows from the foregoing that the response of a polyphase directional relay will be the same for a given type of fault whether or not there is a wye-delta or delta-wye power transformer between the relay and the fault. Although such a power transformer causes an angular shift in the phase-sequence quantities, the positive-phase quantities being shifted in one direction and the negative-phase-sequence quantities in the other, the interacting currents and voltages of the same phase sequence are shifted in the same direction and by the same amount. Hence, there is no net effect aside from the current-limiting-impedance effect of the power transformer itself.

RESPONSE OF SINGLE-PHASE DIRECTIONAL RELAYS TO SHORT CIRCUITS

The impedances seen by single-phase directional relays for different kinds of faults can be constructed by the methods used for distance relays. Reference 10 is an excellent contribution in this respect. Unfortunately the torques of single-phase relays cannot be expressed as simply as for polyphase relays, in terms of positive- and negative-phase-sequence volt-amperes. In other words, simple generalizations cannot be made for single-phase directional relays. It is more fruitful to examine the types of application where misoperation is known to be most likely. This will be done later when we consider the application of directional relays for transmission-line protection.

PHASE-SEQUENCE FILTERS

It is sometimes desirable to operate protective-relaying equipment from a particular phase-sequence component of the three-phase system currents or voltages. Although the existence of phasesequence components may be considered a mathematical concept, it is possible, nevertheless, to separate out of the three-phase currents or voltages actual quantities that are directly proportional to any of the phase-sequence components. The clue to the method by which this can be done is given by the three following equations from symmetrical-component theory:

$$I_{a1} = \frac{1}{3}(I_a + aI_b + a^2I_c)$$

$$I_{a2} = \frac{1}{3}(I_a + a^2I_b + aI_c)$$

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c)$$

These equations give the phase-sequence components of the current in phase *a* in terms of the actual three-phase currents. Knowing the phase-sequence components for phase *a*, we can immediately write down the components for the other two phases. The components of voltage are similarly expressed.

A phase-sequence filter does electrically what these three equations describe graphically. We can quickly dismiss the problem of obtaining the zero-phase-sequence component because we have already seen in Chapter 7 that the current in the neutral of wye-connected CT's is 3 times the zero-phase-sequence component. To obtain a quantity proportional to the positive-phase-sequence component of the current in phase *a*, we must devise a network that (1) shifts I_b 120° counterclockwise, (2) shifts I_c 240° counterclockwise, and (3) adds I_a vectorially to the vector sum of the other two shifted quantities. I_{a2} can be similarly obtained except for the amount of shift in I_b and I_c. Since we are seeking quantities that are only *proportional* to the actual phase-sequence quantities, we shall not be concerned by changes in magnitude of the shifted quantities so long as all three are compensated alike for magnitude changes in any of them. We should remember also that a 120° shift of a quantity in the counterclockwise direction is the same as reversing the quantity and shifting it 60° in the clockwise direction, etc.

Before considering actual networks that have been used for obtaining the various phase-sequence quantities, let us first see some other relations, derived from those already given, that show some other manipulations of the actual currents for getting the desired components. The manipulations indicated by these relations are used in some filters where a zero-phase-sequence quantity does not exist or where it has first been subtracted from the actual quantities (in other words, where $I_a + I_b + Ic = 0$). One combination is:

$$I_{a1} = \frac{1-a^2}{3}(I_b - I_c) - I_b$$

$$I_{a2} = \frac{1-a}{3}(I_b - I_c) - I_b$$

Another combination is:

$$I_{a1} = \frac{1-a^2}{3}(I_a - a^2 I_b)$$

$$I_{a2} = \frac{1-a}{3}(I_a - a I_b)$$

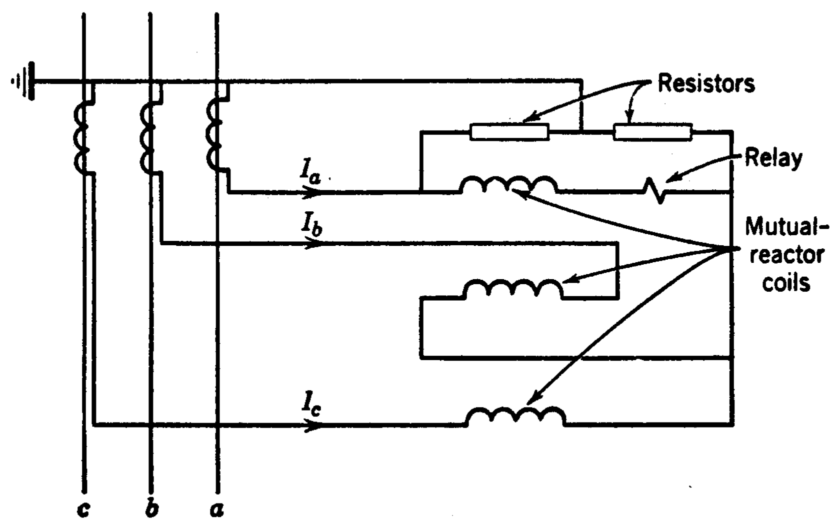

Fig. 22. A positive-phase-sequence-current filter and relay.

Still another combination is:

$$I_{a1} = \frac{a-a^2}{3}(I_b - a I_a)$$

$$I_{a2} = \frac{a^2-a}{3}(I_b - a^2 I_a)$$

Figure 22 shows a positive-phase-sequence-current filter that has been used;[11] by interchanging the leads in which I_b and I_c flow, the filter becomes a negative-phase-sequence-current filter. A filter that provides a quantity proportional to the negative-phase-sequence component alone, or the negative plus an adjustable proportion of the zero, is shown in Fig. 23;[12] by interchanging the leads in which I_b and I_c flow, the filter becomes a positive-plus-zero-phase-sequence-current filter.

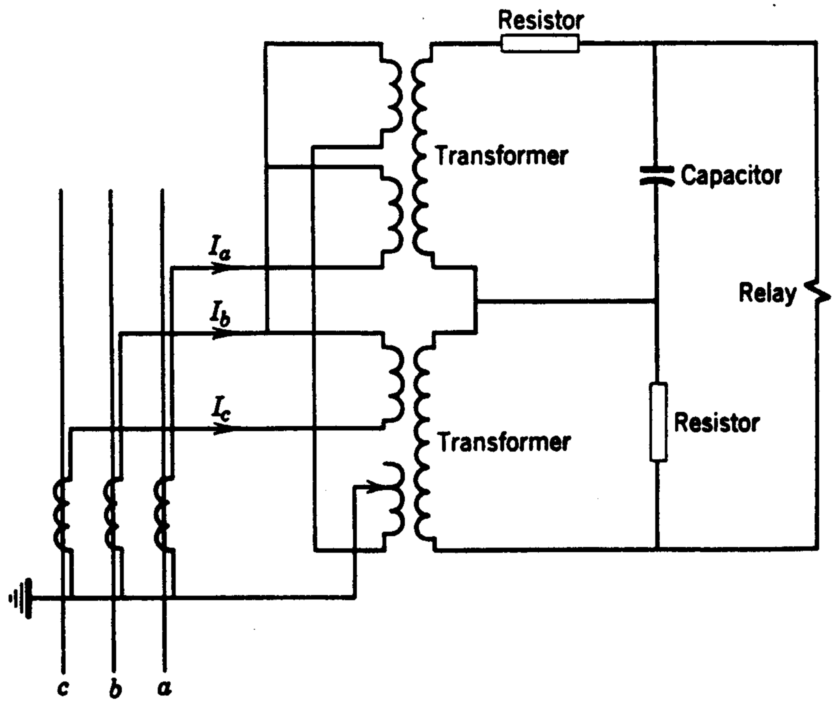

Fig. 23. A combination negative- and zero-phase-sequence-current filter and relay.

The references given for the filters of Figs. 22 and 23 contain proof of the capabilities of the filters. The purpose here is not to consider details of filter design but merely to indicate the technique involved. Variations on the filters shown are possible. Excellent descriptions of the many possible types of filters are given in References 1 and 13 of the Bibliography. These books show voltage as well as current filters.

A consideration that should be observed in the use of phase-sequence filters is the effect of saturation in any of the filter coil elements on the segregating ability of the filter. Also, frequency variations and harmonics of the input currents will affect the output.[14] It will be appreciated that the actual process of shifting quantities and combining them, as indicated by symmetrical-component theory, requires a high degree of precision if the derived quantity is to be truly representative of the desired quantity.

PROBLEMS

1. Given the system shown in Fig. 24, including the transmission-line sections *AB* and *BC*, and mho-type phase distance relays for protecting these lines. On an *R-X* diagram, show quantitatively the characteristics of the three mho units of each relay for providing primary

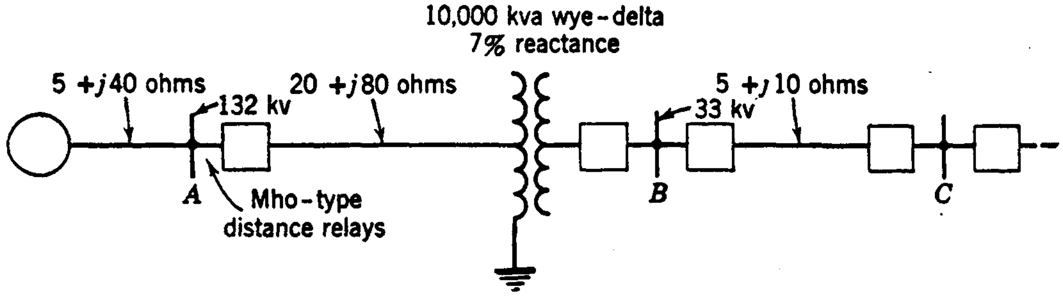

Fig. 24. Illustration for Problem 1.

protection for the line *AB* and back-up protection for the line *BC*. Assume that the centers of the mho characteristics are at 60° lag on the *R-X* diagram.

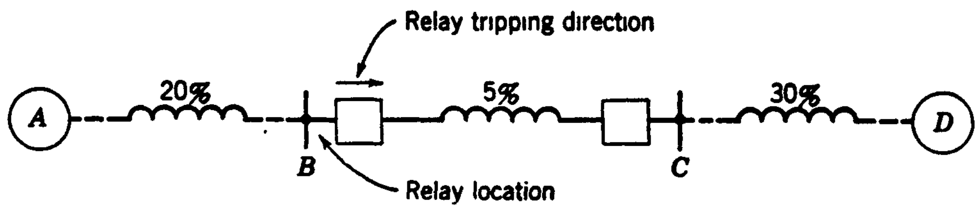

Fig. 25. Illustration for Problem 2.

2. Given a system and a distance relay located as shown in Fig. 25. Assume that the relay is a mho type having distance and time settings as follows:

Zone	Distance Setting	Time Setting
1st	90% of line *BC*	High speed
2nd	120% of line *BC*	0.3 sec
3rd	300% of line *BC*	0.5 sec

Assume also that the centers of the mho circles lie at 60° Iag on the *R-X* diagram. The values on the system diagram are percent impedance on a given kva base. Assume that the *X/R* ratio of each impedance is tan 60°. As viewed from the relay location, the system is transmitting synchronizing power equal to base kva at base voltage over the line *BC* toward *D* at 95% lagging PF, when generator *A* starts to lose synchronism with generator *D* by speeding up with respect to *D*. Assume that the rate of slip is constant at 0.2 slip-cycle per second. Will the relay operate to trip its breaker? Illustrate by showing the relay and the loss-of-synchronism characteristics on an *R-X* diagram.

3. Given a polyphase directional relay, the three elements of which use the combinations of current and voltage shown on p. 192.

Element No.	I	V
1	$I_a - I_b$	$V_{bc} - V_{ac}$
2	$I_b - I_c$	$V_{ca} - V_{ba}$
3	$I_c - I_a$	$V_{ab} - V_{cb}$

Write the torque equations in terms of the positive- and negative-phase-sequence volt-amperes. What do you think of this as a possible connection for such relays?

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